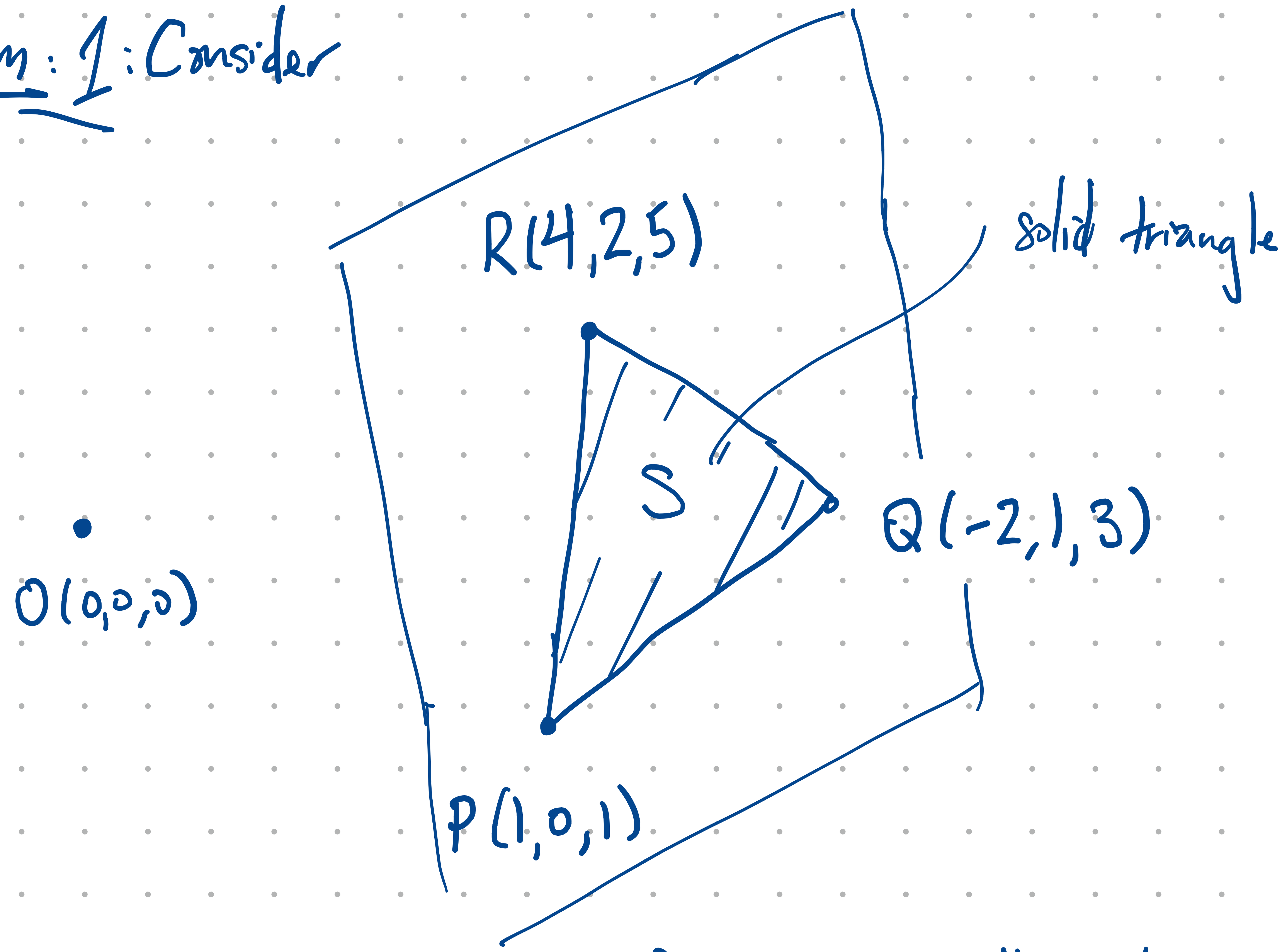
Problem 1: Consider

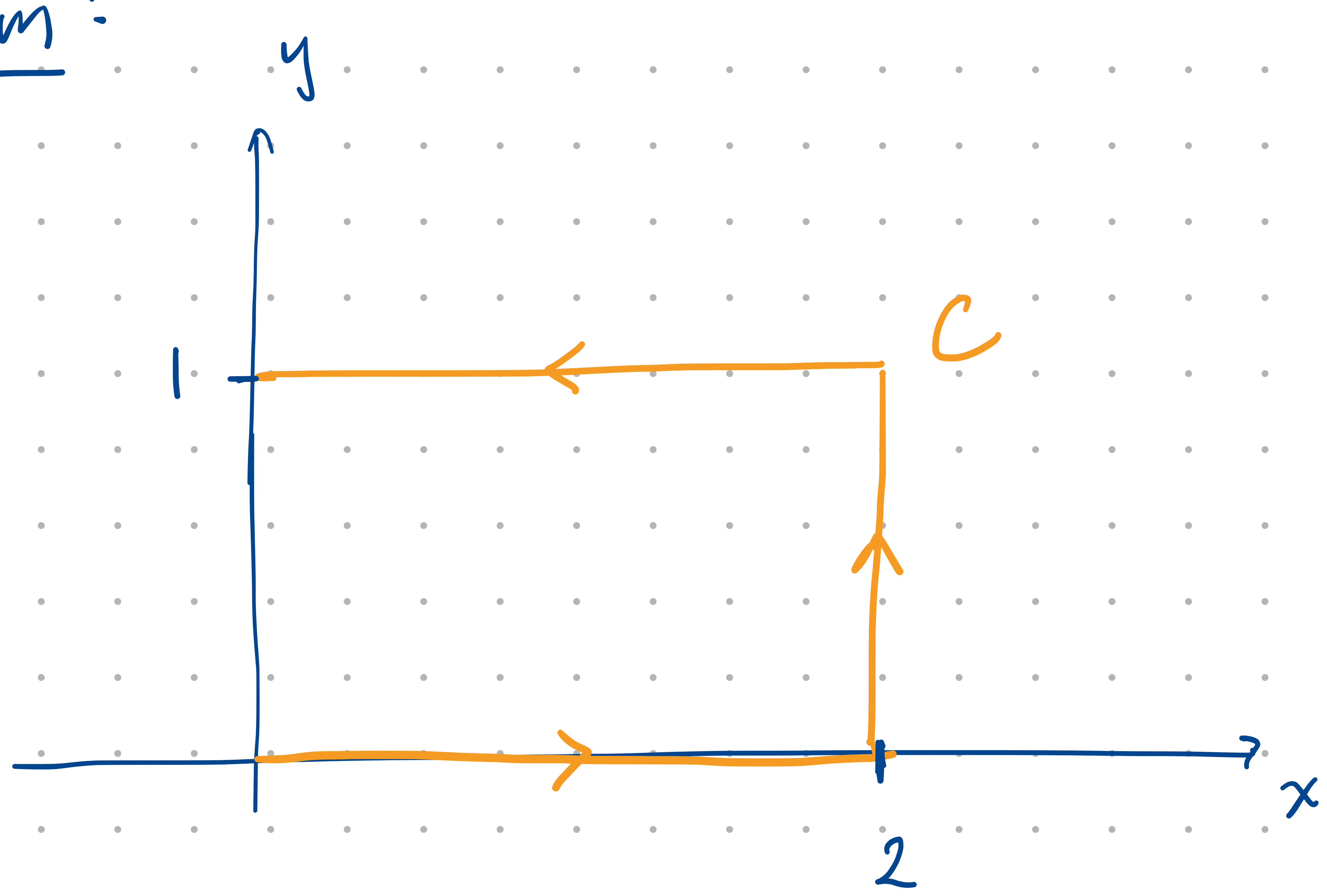


Compute the flux of $\langle x, y, z \rangle$ through S in the direction away from the origin.

a) Do this by first parametrizing S, and then setting up the integral directly

b) Can you figure out a way to solve this using the Divergence Thm?

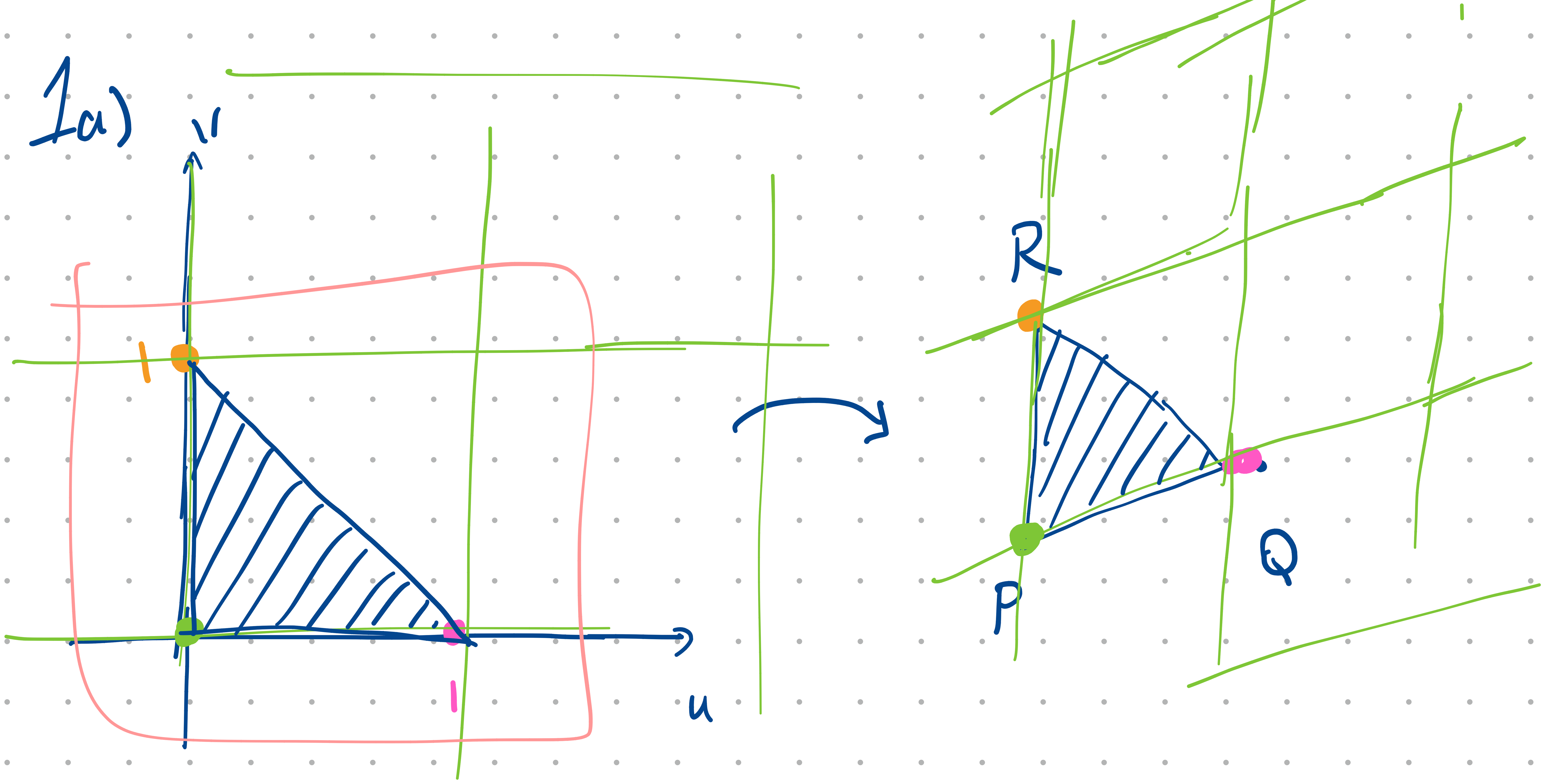
Problem:



Compute $\int_C \langle 2xy, x^2 + x \rangle \cdot d\vec{r}$ in as many different ways as you can think of.

e.g. direct computation, FTLI, Green's Thm...

1a)



$$\vec{r}(u,v) = \langle 1,0,1\rangle + u\underbrace{\langle -3,1,2\rangle}_{\overrightarrow{PQ}} + v\underbrace{\langle 3,2,4\rangle}_{\overrightarrow{PR}}$$

$$= \boxed{\langle 1-3u+3v,\ u+2v,\ 1+2u+4v\rangle}$$

Alternative approach: What plane is the triangle in?

$$\vec{n} = \langle -3,1,2\rangle \times \langle 3,2,4\rangle = \langle 0,18,-9\rangle$$

$$\vec{n}\cdot(\vec{r}-\vec{r}_0) = 0$$

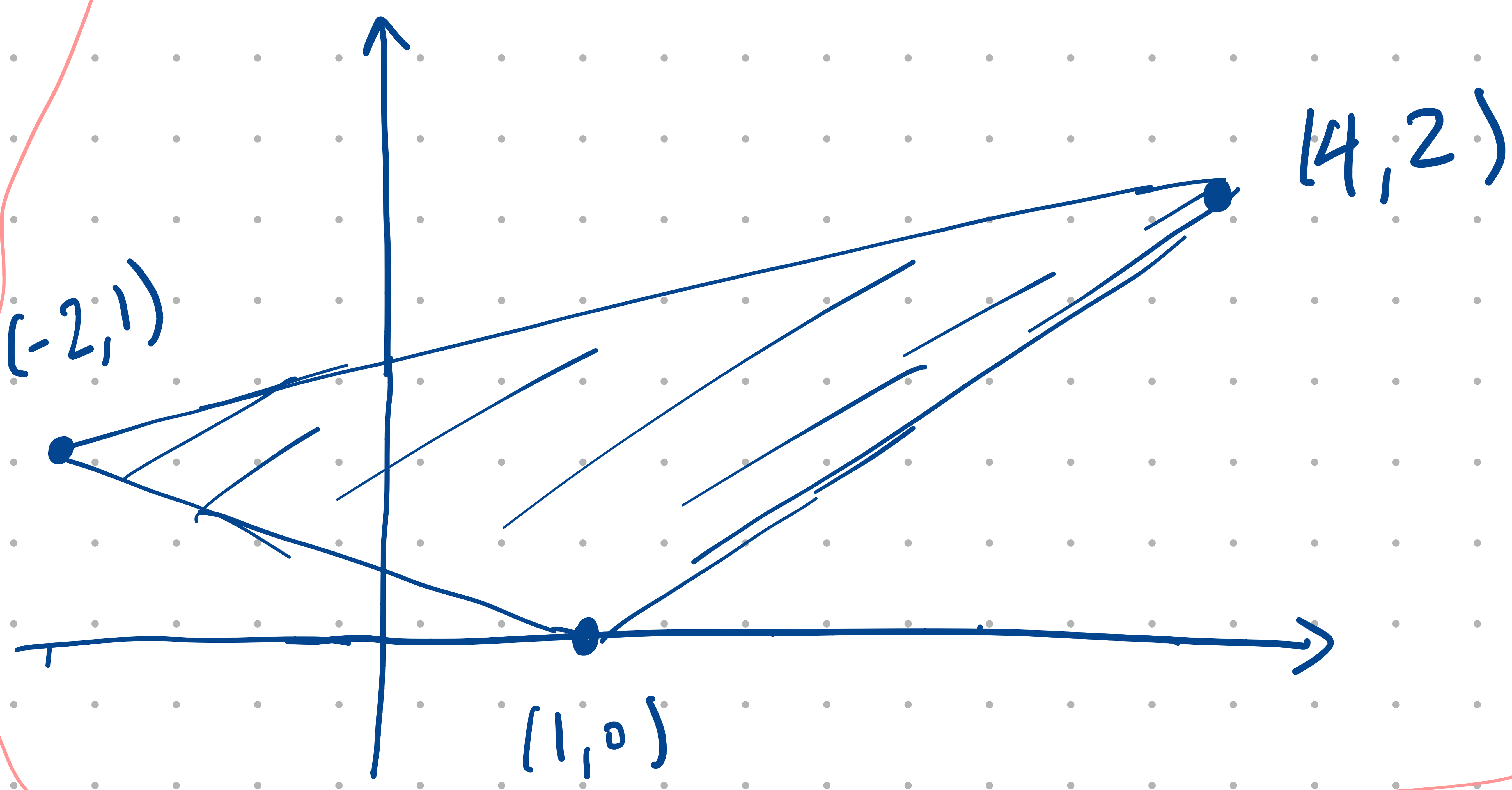
$$18(y-0) + (-9)(z-1) = 0$$

$$18y - 9z + 9 = 0$$

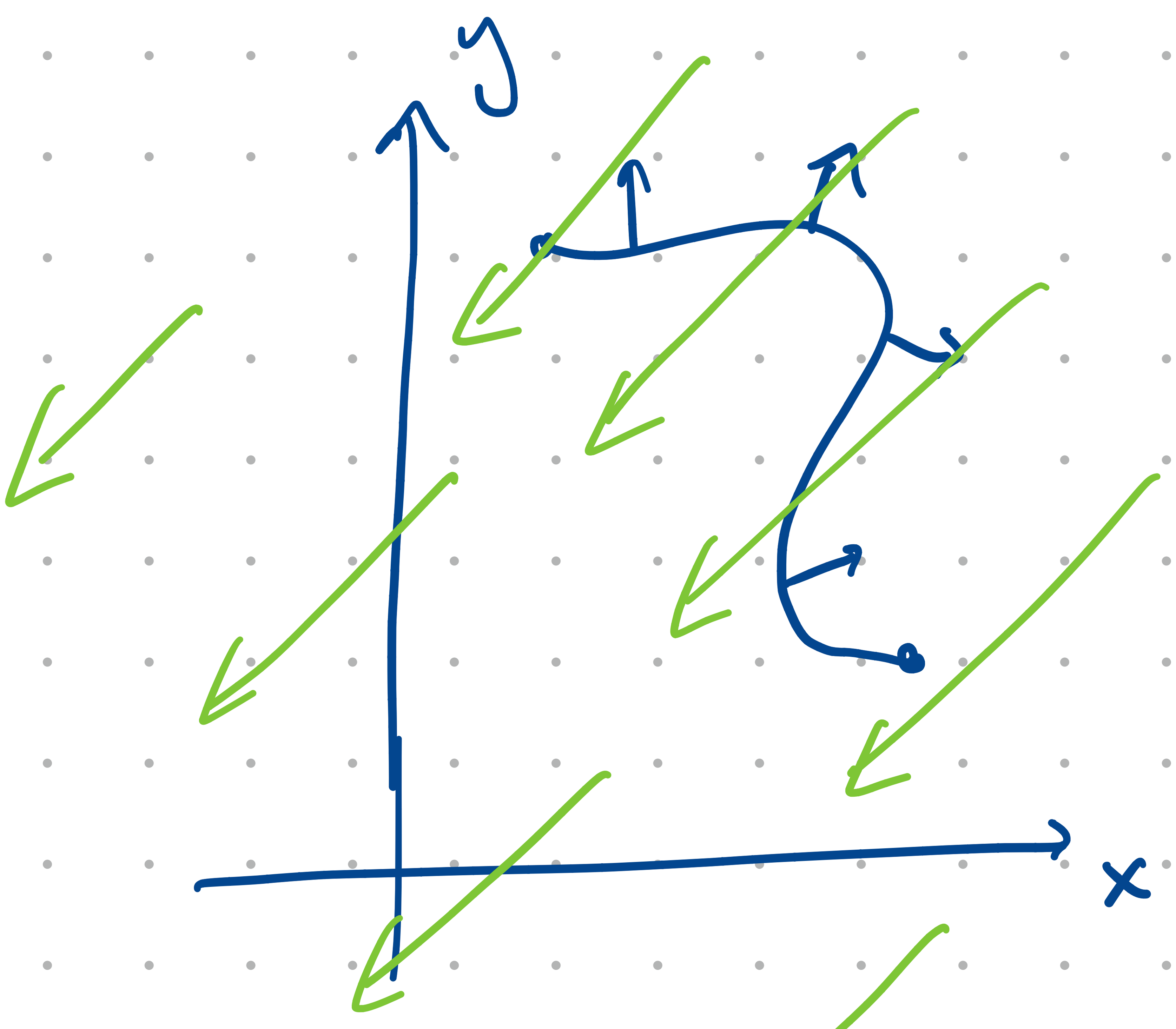
$$2y - z + 1 = 0$$

so a parametrization could be

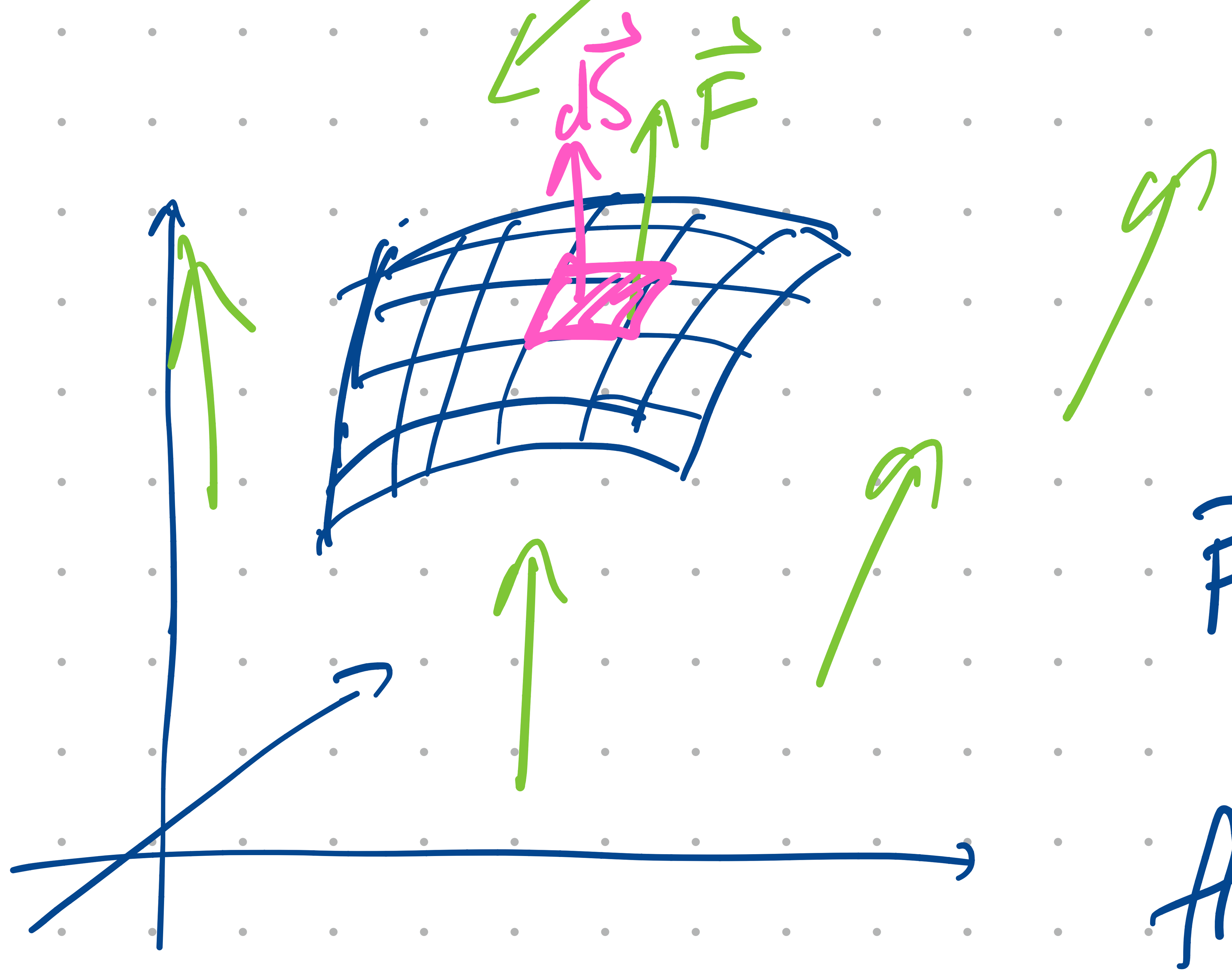
$$\vec{r}(x,y) = \langle x, y, 2y+1 \rangle$$



Comment on flux: it is a measurement of "how much" of your vector field is flowing "through" an object.



See end of §16.5 for flux in 2-dimensions.



$\vec{F} \cdot d\vec{S}$ is the infinitesimal amount flowing through the tiny patch

So altogether one integrates over the surface

$$\iint_{\text{surface}} \vec{F} \cdot d\vec{S}$$

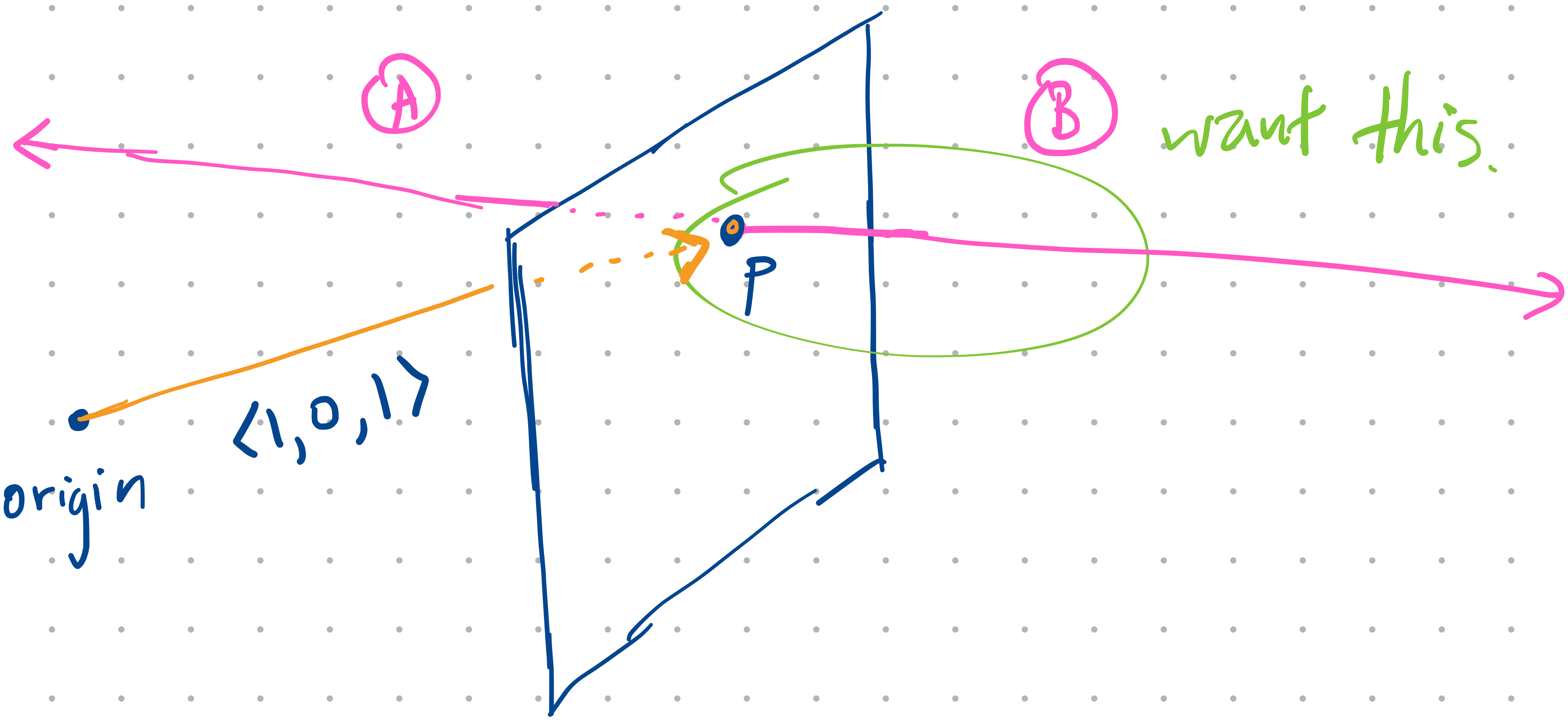
Back to the problem, (using the first parametrization):

$\vec{r}_u \times \vec{r}_v$ or $\vec{r}_v \times \vec{r}_u$ (which one?)

compute this, find

$\langle 0, 18, -9 \rangle$

is this Ⓐ or Ⓑ?



$\langle 1,0,1 \rangle \cdot \langle 0,18,-9 \rangle = -9 < 0$

so the angle between them is obtuse.

So $\langle 0,18,-9 \rangle$ is Ⓐ

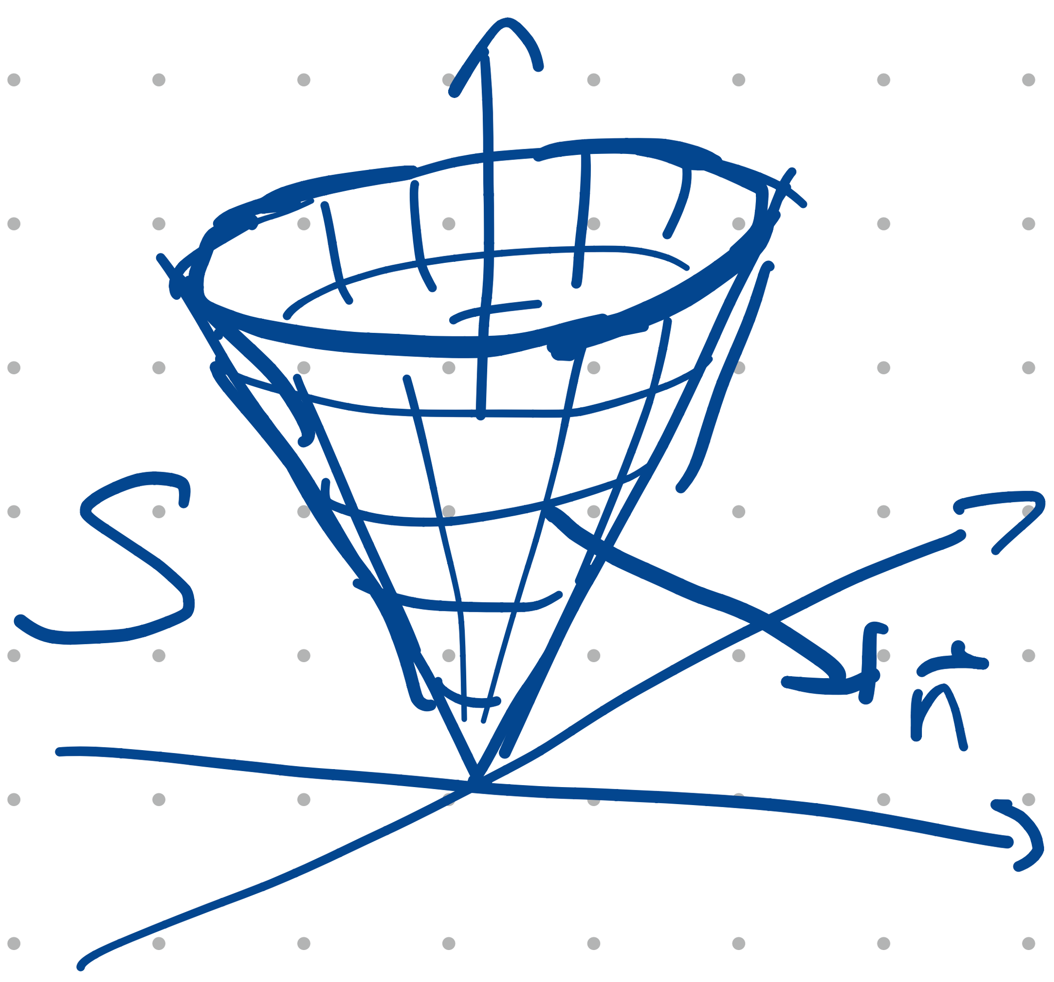
and I actually want $\langle 0, -18, 9\rangle$

$$\int_0^1 \int_0^{1-u} \langle 1-3u+3v, u+2v, 1+2u+4v\rangle \cdot \langle 0,-18,9\rangle \, dv \, du$$

$$= \ldots\ldots$$

$$= \boxed{9/2}$$

Before we move on to 1b), consider a question like...



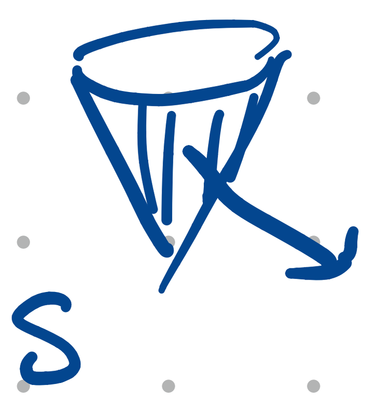
some vector field

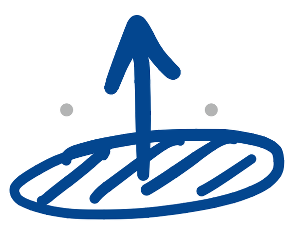
$\vec{F}$

Asked to compute $\iint_S \vec F \cdot d\vec S$

Could compute directly, or use div thm:

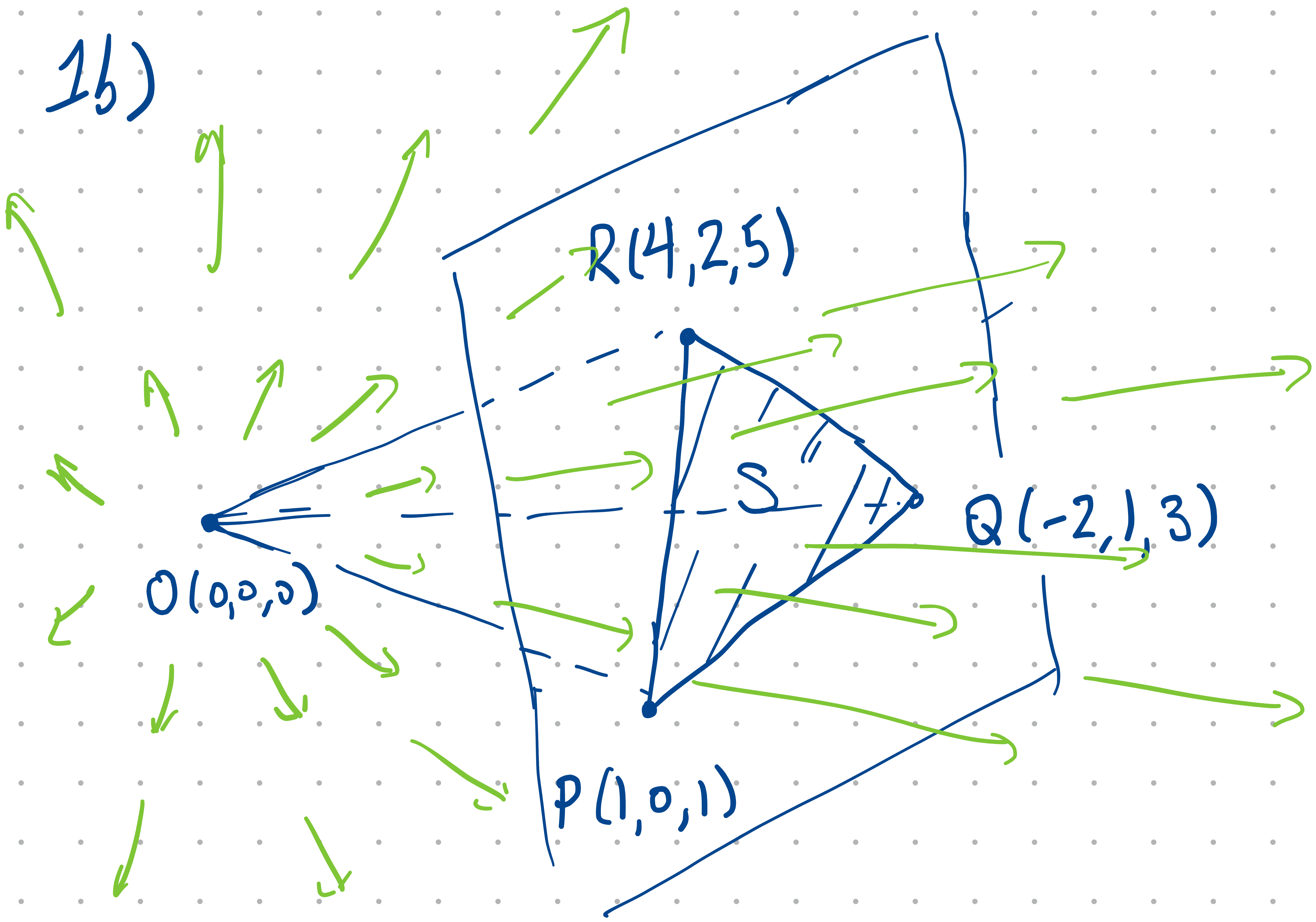
$$\iint \vec F \cdot d\vec S = \iiint \operatorname{div} \vec F \, dV - \iint \vec F \cdot d\vec S$$





The moral is: even is the surface isn't closed, sometimes it may be easy to close it and then use the divergence thm.

1b)



solid tetrahedron OPQR (call it E)

Boundary of E is 4 triangles (S is one of them).

$$\iiint_E \underbrace{\operatorname{div}\langle x,y,z\rangle}_{3}\, dV = \iint_S \langle x,y,z\rangle\cdot d\vec{S} + \cancel{\iint_{\text{other three triangles}} \langle x,y,z\rangle\cdot d\vec{S}} \to 0$$

$$\| $$

$$3\iiint_E dV = 3\,\text{Vol}(E).$$

why is this equal to 0?

(Examine picture: the vec. field is parallel to the other three faces)

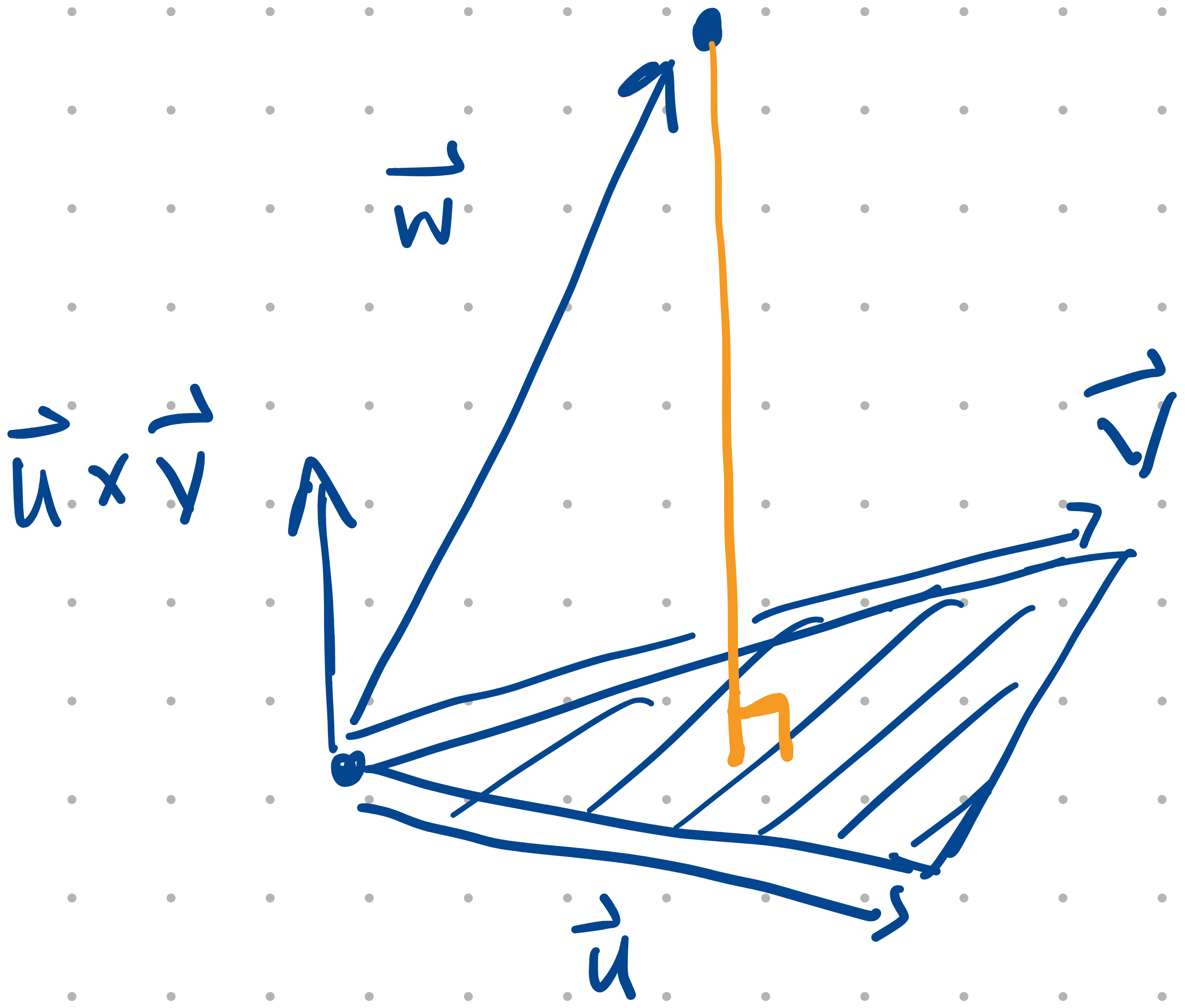
So need to compute

$$3\cdot \text{Vol}(\text{tetrahedron } OPQR).$$

$$= 3\cdot\left(\frac{1}{3}(\text{Area of base})(\text{height})\right)$$

$$= (\text{Area of base})(\text{height}) = \left(\frac{1}{2}\cancel{|\vec{u}\times\vec{v}|}\right)\cdot\left(\frac{|\vec{w}\cdot(\vec{u}\times\vec{v})|}{\cancel{|\vec{u}\times\vec{v}|}}\right)$$

abs val. of scalar proj, i.e. $\left|\text{comp}_{\vec{u}\times\vec{v}}\,\vec{w}\right|$



$$= \frac{1}{2}\left|\vec{w}\cdot(\vec{u}\times\vec{v})\right|$$